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No "big trips" for the universe

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Abstract

It has been claimed in several papers that a phantom energy-dominated universe can undergo a "big trip", i.e., tunneling through a wormhole that grows faster than the cosmic substratum due to the accretion of phantom energy, and will reappear on the other mouth of the wormhole. We show that such claims are unfounded and contradict the Einstein equations.

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The modern picture of the accelerating universe has led to many dark energy models accounting for the observed acceleration. While observational support from distant supernovae of type Ia [1] is still marginal, models incorporating a phantom energy component (i.e., a form of energy with pressure $P < -\rho$, where ρ is the energy density) have received the attention of many authors [2]. Such models may exhibit a big rip, a singularity occurring at a finite time in the future at which the scale factor, the energy density, and the pressure diverge [3]. In the context of such models it has recently been proposed that the universe could avoid the big rip by tunneling though a wormhole contained in it, disappearing through one mouth and reappearing from the other mouth [4], a process dubbed "big trip". The mechanism by which the big trip would be achieved would be the catastrophic growth of a wormhole by accretion of phantom energy at a speed larger than the expansion rate of the cosmic substratum. In Ref. [5] it was shown that the original claim of such a possibility is unfounded, not being based on actual calculations, and explicit exact solutions were presented describing wormholes embedded in a Friedmann–Lemaitre–Robertson–Walker (hereafter FLRW) universe and accreting phantom energy from the cosmic fluid. While the universe approaches the big rip, these rather general classes of wormhole solutions end up expanding and becoming comoving, even if they initially expand with arbitrary velocity relative to the cosmic fluid [5]. Another solution of this kind was found in Ref. [6]. However, more recent claims that the big trip is possible have appeared in the literature ([7,8]—see also [9–12]); these are again not substantiated by calculations and reflect more the wishful thinking of these authors than real results. Here we want to settle the issue of the big trip.

In his recent claim of a big trip [7], as well as in the first paper containing such a claim [4], Gonzalez-Diaz uses spherically symmetric *static* metrics to deduce the possibility of the big trip. Ref. [7] is structured as a reply to potential or published objections to the possibility of the big trip. Aside from the difficulty for stable macroscopic wormholes to exist, the first and in our view, the most serious, objection [5] is that a static metric can not possibly describe an extremely rapid, catastrophic accretion process. Ref. [7] is flawed in this regard, as is shown in the following. This Letter begins with the static Morris–Thorne wormhole metric [13] with zero shift function (i.e., $g_{00} = -1$)

$$ds^2 = -dt^2 + \frac{1}{1 - K(r)/r} dr^2 + r^2 d\Omega_2^2, \qquad (1)$$

where $d\Omega_2^2$ is the metric on the unit two-sphere. The Einstein equations for a spherically symmetric metric of the form

$$ds^2 = -e^{\nu(t,r)} dt^2 + e^{\lambda(t,r)} dr^2 + r^2 d\Omega_2^2 \qquad (2)$$

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assume the form [14]

$$e^{-\lambda}\left(\frac{\nu'}{r}+\frac{1}{r^2}\right)-\frac{1}{r^2}=8\pi G T_1^1, \tag{3}$$

$$\frac{e^{-\lambda}}{2}\left(\nu''+\frac{(\nu')^2}{2}+\frac{\nu'-\lambda'}{r}-\frac{\nu'\lambda'}{2}\right)-\frac{e^{-\nu}}{2}\left(\ddot{\lambda}+\frac{(\dot{\lambda})^2}{2}-\frac{\dot{\lambda}\dot{\nu}}{2}\right)$$
$$=8\pi G T_2^2=8\pi G T_3^3, \tag{4}$$

$$e^{-\lambda}\left(\frac{1}{r^2}-\frac{\lambda'}{r}\right)-\frac{1}{r^2}=8\pi G T_0^0, \tag{5}$$

$$e^{-\lambda}\dot{\lambda}=8\pi G r T_0^1, \tag{6}$$

where T_{ab} is the energy–momentum tensor, $\dot{} \equiv \partial/\partial t$, and $' \equiv \partial/\partial r$. For the metric (1), Eq. (6) is equivalent to

$$\dot{K}=8\pi G(r-K)^2 T_0^1. \tag{7}$$

Therefore, a static metric which corresponds to $\dot{K}=0$ automatically implies $T_0^1=0$ and vice versa. There is no radial energy flux and no accretion of phantom energy onto the wormhole, let alone accretion of the entire universe! The use of equations derived from a static metric cannot be justified for extreme accretion regimes. Historically, this point was not missed by McVittie when he introduced in his 1933 paper [15] the well known McVittie solution

$$ds^2=-\left(\frac{1-\frac{m(t)}{2r}}{1+\frac{m(t)}{2r}}\right)^2 dt^2$$
$$+a^2(t)\left(1+\frac{m(t)}{2r}\right)^4\left(dr^2+r^2\,d\Omega_2^2\right) \tag{8}$$

describing a spherically symmetric central object embedded in a FLRW cosmological background in isotropic coordinates (here we limit ourselves to the spatially flat case relevant in the context of Refs. [4,7–10,12]). The central mass satisfies

$$\frac{\dot{m}}{m}=-\frac{\dot{a}}{a}, \tag{9}$$

which follows from the equation ${T_0}^1=0$, which in turn is derived from the explicit assumption that the central object does not accrete the cosmic fluid (assumption *e*) of Ref. [15]). This solution of the Einstein equations shows that, in fact, in order to have accretion not only the metric must be time-dependent, but also the energy–momentum tensor must have a mixed component ${T_0}^1$ describing the radial energy flow onto the central object, and missing in a static metric such as (1). Then, Eq. (9) will contain an extra term describing this radial flow. While no change in the mass of the wormhole, and therefore a static metric, could be reasonable approximations for slow accretion rates, Gonzalez-Diaz claims that their use is justified in the extreme accretion regime that he wants to describe, which contradicts the Einstein equation (6). He states that "… even though the starting metric is static, the energy stored in the wormhole must change with time by virtue of dark energy accretion…" [7]: it is impossible to have T_0^0, T_1^1, T_2^2, and T_3^3 time-dependent on the right-hand sides of Eqs. (3)–(5) and simultaneously have the left-hand sides (containing λ and its derivatives) time-independent. Refs. [4,7,8,10] do not propose a single solution of the Einstein equations but rather try to deduce the growth rate of the wormhole mass by modifying formulas valid for static solutions in an inconsistent way, as shown in the following.

The discussion of [7] is based on the covariant conservation equation $\nabla^b T_{ab}=0$ taken from Ref. [16] (hereafter "BDE") and replacing the final result of this reference containing a $K(r)$ with a time-dependent $K(t,r)$. Now, the equations of [16] were derived for:

(1) a T_{ab} describing a non-gravitating *test fluid*, which can not represent the cosmic fluid causing the expansion of the universe and the big rip;
(2) the static *Schwarzschild metric* corresponding to *vacuum* (consistent with assumption (1)).

One needs instead a metric describing simultaneously the central object (wormhole) *and* the cosmological background universe in which it is embedded, à la McVittie. The conservation equation $\nabla^b T_{ab}=0$ used in [7] refers to the Schwarzschild metric describing only a central black hole. While the last metric presented in [7] could in principle achieve the goal, no calculations pertaining to it are presented. Now, the first conservation equation of [7] corresponds to Eq. (3) of [16] and can only be obtained in *vacuum* (i.e., with T_{ab} describing a non-gravitating test fluid that is accreted) and the Schwarzschild metric. In fact, using the general spherically symmetric metric (2) the conservation equation projected on the u^a direction, $u^a\nabla_b {T_a}^b=0$ becomes

$$\frac{\rho'}{P+\rho}+\frac{u^t}{u^r}\frac{\dot{\rho}}{P+\rho}+\frac{u^t}{u^r}\left(\frac{\dot{\nu}+\dot{\lambda}}{2}\right)+\frac{\partial_t u^t}{u^r}$$
$$+\frac{\lambda'+\nu'}{2}+\frac{2}{r}+\partial_r\left(\ln u^r\right)=0.$$

This can be simplified to Eq. (3) of BDE used in [7] only under the assumptions of vacuum, $\partial_t u^a=0$, $\dot{\rho}=0$, and the metric is then forced to be Schwarzschild. Then, $\lambda=-\nu$ [14] and we obtain

$$r^2 u^r e^{\int \frac{d\rho}{P+\rho}}=C$$

(Eq. (3) of BDE). Ref. [7] neglects the derivatives $\dot{\nu}$, $\dot{\lambda}$, ν', λ', $\dot{\rho}$ and u^t and simply replaces $K(r)$ with $K(t,r)$ in the result valid for a vacuum static metric, and the metric proposed does not solve the Einstein equations.

Second, the conservation equation $\nabla^b T_{ab}=0$ alone is certainly not sufficient to determine the metric; it determines the dynamics of (test or gravitating) matter if the metric is given. This is exactly the point of view of the authors of Ref. [16], who assume a Schwarzschild metric and compute the accretion rate of a *test fluid on the Schwarzschild black hole*. Gonzalez-Diaz takes the conservation equation from [16] and tries to determine the metric from $\nabla^b T_{ab}=0$ alone, which is impossible. In reality the metric is still Schwarzschild but in [7] it is proposed to alter it a posteriori by adding time dependence in $K(r)$. However this incorrect procedure neglects most of the relevant terms in the Einstein equations.

A general proof of the non-existence of the big trip is, of course, impossible, because one does not know the most general solution of the Einstein equations representing a wormhole, even for spherical symmetry. This is due to the failure of Birkhoff's theorem in non-vacuum, non-asymptotically flat space-times and to the lack of a precise definition of wormhole.[1] We can, however, restrict to metrics of the form (1) with K promoted to a function of *both* t and r, which is considered in [7]. Then, $\nu \equiv 0$ and $e^{\lambda} = (1 - \frac{K}{r})^{-1}$. Let us consider a perfect fluid with stress-energy tensor $T_{ab} = (P + \rho)u_a u_b + P g_{ab}$, where $u^a = (u^0, u, 0, 0)$ is the fluid four-velocity with radial component $u \equiv u^r < 0$ describing inflow onto the wormhole. The normalization $u^c u_c = -1$ yields $u^0 = e^{\lambda}|u|$. The *proper* velocity of the fluid is

$$v \equiv \frac{dr_{\text{phys}}}{d\tau} = \left(1 - \frac{K}{r}\right)^{-1} \frac{dr}{d\tau} = \frac{u}{1 - K/r}, \tag{10}$$

where $dr_{\text{phys}} = \sqrt{g_{rr}}\, dr$ is the proper distance in the radial direction and $\tau = t$ is the proper time. By combining Eqs. (3) and (6) and the expression of v one obtains

$$\frac{\lambda' e^{\lambda}}{e^{2\lambda} - 1} = 8\pi G r |P + \rho| v^2. \tag{11}$$

After straightforward manipulations of the left-hand side, Eq. (11) is rewritten as

$$\frac{d}{dr}\left[\ln\left(\frac{e^{\lambda} + 1}{|e^{\lambda} - 1|}\right)\right] = 16\pi G r |P + \rho| v^2. \tag{12}$$

Assuming, as done in [7], that $(P + \rho) = (w + 1)\rho = (w + 1)\rho_0 a^{-3(w+1)}$ in a universe dominated by phantom energy with equation of state $P = w\rho$ ($w < -1$) heading toward the big rip, the right-hand side diverges unless $v \sim a^{3(w+1)/2}$, which implies that

$$\ln\left(\frac{e^{\lambda} + 1}{|e^{\lambda} - 1|}\right) = 16\pi G \int dr\, r |P + \rho| v^2 \tag{13}$$

diverges as the big rip is approached, or that $K(t, r) \to 0$ for all values of r in this limit; but then the function K disappears from the metric (1), which becomes the Minkowski metric! This conundrum is avoided only if $v \sim a^{3(w+1)/2}$, which guarantees that the right-hand side of Eq. (11) stays finite. But the vanishing of the proper radial velocity of the fluid means that accretion stops asymptotically near the big rip, and the big trip is again prevented.

In addition to these quantitative arguments and to the explicit solutions constructed in Refs. [5,6] we remark that, from the purely conceptual point of view, it is hard to give a meaning to the idea of an entire universe disappearing into one of its parts contained within it. A vague argument invoking a multiverse, sketched in Ref. [7], fails to address this problem, as the author of [7] himself seems to be aware of.

To summarize: here we show that the proposed metrics with the big trip property are not solutions of the Einstein equations. To study the possibility of a big trip one must examine metrics describing simultaneously the wormhole and the universe in which it is embedded (with T_{ab} a gravitating fluid) and solve the Einstein equations consistently. This is what is done in Refs. [5,6]. In spite of the fact that rather large classes of solutions were found, none of them has the big trip property. We do not claim that these are *all* the possible solutions for a wormhole embedded in a phantom energy universe: they are just all the solutions known to date. Ref. [7] and the companion papers propose guesses on how the equations and the solutions for a big trip would look like, and these guesses are incorrect. It has not been shown to date that the big trip of the universe is possible or has physical meaning.

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